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Acceleration of a relativistic plasma by radiation pressure

E. S. Phinney *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

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Summary. The greatly enhanced radiation pressure force felt by a relativistic plasma is accompanied by catastrophic Compton cooling and only in extreme conditions can it lead to acceleration to relativistic bulk velocities. We solve the equations of motion in the optimal case and find that the efficiency of acceleration is typically < 1 per cent (the energy lost being given to the scattered photons). The complicating effects of expansion of the plasma, finite source size and scattering above the Klein–Nishina limit are described. We end with a short list of situations in which the phenomenon may nevertheless be of importance.

1 Introduction

O'Dell (1981) has recently pointed out that a relativistic plasma in an anisotropic radiation field feels a much larger radiation pressure than a non-relativistic plasma of the same inertial mass and that it drives itself away from the radiation source with energy derived largely from its own internal energy rather than that of the incident photons. He suggests that this 'rocket' effect may be important in the dynamics of compact radio sources. While the modified Eddington limit is certainly important in determining the *statics* of matter and radiation in relativistic potential wells, we here demonstrate that even in the best of circumstances, the 'rocket' is not a particularly efficient way to accelerate plasma to relativistic bulk velocities (as are, for example, inferred in rapidly variable radio sources or sources expanding superluminally on VLBI scales). This is because every photon 'batted' back towards the source carries away on average twice as much energy as momentum (we employ geometric units, $G = c = 1$, throughout), so the plasma is cooled faster than it is accelerated.

We derive the basic equations in Section 2, solve them in Section 3 in the idealized (but best of all possible) cases where the only energy losses and momentum transfers are due to the Compton process, and discuss in Section 4 complications and the conditions in which the 'rocket' can operate. Section 5 attempts to rekindle the reader's enthusiasm.

2 Basic equations

The reason for the rocket effect is easily understood. Suppose some scatterers with cross-section σ, mass m_e and typical energy $\gamma_r m_e \gg m_e$, are all coupled together (see Section 4) and can maintain a momentum distribution that is (roughly) isotropic in some frame. Each scatterer 'bats' photons with energies (and momenta!) boosted by a factor $\sim \gamma_r^2$ into a cone of half-angle $\sim 1/\gamma_r$ having an axis along the direction of the scatterer's motion. However, those scatterers moving towards the source of photons bat about twice as many photons to energies about twice as high as do those moving away from the source. We therefore expect the average rate of momentum transfer to the plasma to be $dp/dt \sim \sigma F \gamma_r^2$ per scatterer. The rocket is driven by the anisotropy of the radiation scattered by the plasma.

The exact result can be derived very easily. Consider a particle at rest that scatters photons of unchanged energies into an angular distribution symmetric front-to-back along the axis defined by a photon's original direction of motion (all true for electrons in the Thomson scattering limit). The equations for the rate of change of its four-momentum are

$$\frac{dp^{0'}}{d\tau} = 0$$

$$\frac{dp^{i'}}{d\tau} = \sigma T^{0'i'}, \tag{1}$$

where τ is proper time, primes denote quantities measured in the particle's rest frame and $T^{\mu\nu}$ is the stress–energy tensor of the radiation field. The only frame-independent equation of motion linear in $T^{\mu\nu}$ that can be constructed from the particle's four-velocity $u^\mu = p^\mu/m_e$, $T^{\mu\nu}$, σ and m_e, and which reduces to equations (1) in the rest frame of the particle is (in a metric of signature + 2)

$$\frac{dp^\mu}{d\tau} = -\sigma(u_\alpha T^{\alpha\mu} + [u_\alpha T^{\alpha\beta} u_\beta]\, u^\mu). \tag{2}$$

Note that this equation is valid in curved space as well as flat.

The equations derived by O'Dell are merely a special case: a plane-parallel (or point source) radiation field propagating in the $\mathbf{e}_1$ direction is described by a stress–energy tensor with all components zero except $T^{00} = T^{01} = T^{10} = T^{11} = F$, the flux. Then putting $u_\mu = (-\gamma_r, \gamma_r\beta_r \cos\theta, u_2, u_3)$ into equation (2), we have

$$\frac{dp^0}{dt} = \frac{1}{\gamma_r}\frac{dp^0}{d\tau} = -\sigma F[-(1-\beta_r\cos\theta) + \gamma_r^2(1-\beta_r\cos\theta)^2]$$

$$\frac{dp^1}{dt} = \frac{1}{\gamma_r}\frac{dp^1}{d\tau} = \sigma F[(1-\beta_r\cos\theta) - \gamma_r^2(1-\beta_r\cos\theta)^2\beta_r\cos\theta] \tag{3}$$

which are O'Dell's equations (6) and (7), before averaging over particles. Averaging (3) over an isotropic momentum distribution gives the average energy loss of, and force on, a scatterer

$$\left\langle\frac{dp^{0'}}{dt'}\right\rangle \equiv \left\langle\frac{dE'}{dt'}\right\rangle = -\sigma F' \frac{4}{3}\langle\gamma_r^2\beta_r^2\rangle \tag{4}$$

$$\left\langle\frac{dp^{1'}}{dt'}\right\rangle = \sigma F'\left(1 + \frac{2}{3}\langle\gamma_r^2\beta_r^2\rangle\right), \tag{5}$$

where primes and subscript r's denote quantities measured in the frame which the momenta are isotropic.

These are O'Dell's equations (9) and (10). Equation (5) has the form expected on the basis of the intuitive argument given earlier, while (4) is identical to the inverse-Compton cooling rate of particles in an isotropic radiation field. (Equation (2) provides the most elegant possible derivation of this familiar result: an isotropic radiation field with energy density U is described by $T^{\mu\nu} = \mathrm{diag}\,(U, U/3, U/3, U/3)$. Putting $u_\mu = (-\gamma, \gamma\boldsymbol{\beta})$ and using $d/dt = 1/\gamma\, d/d\tau$ in (2), we immediately have $dp^0/dt = -(4/3)\,\sigma\gamma^2\beta^2 U$.)

3 Acceleration and cooling in the ideal case

There is some evidence that relativistic plasmas may be present near strong, effectively point-sources of radiation in many compact radio sources (Wardle 1977) and active nuclei. There is also evidence for relativistic bulk motion in these same sources (rapidly varying radio fluxes, superluminal expansion on VLBI scales, *cf.* Pearson *et al.* 1981). One is thus led to inquire how efficiently the Compton rocket can convert relativistic random motions into relativistic bulk motion. We defer to Section 4 a discussion of complications, competing processes and the conditions required for operation of the rocket, and here solve the problem in the optimal case where the only energy losses and accelerations are those due to the Compton process itself – i.e. we consider the motion of a relativistic plasma confined to a transparent cage (to avoid the cooling and acceleration due to free expansion) and which emits no radiation of its own (synchrotron, bremsstrahlung, annihilation, etc.). It is placed at a distance $R_0 = \omega_0 M$ from a point source of radiation with luminosity $L = lL_{\mathrm{Edd}} = l\,(4\pi M m_{\mathrm{p}}/\sigma_{\mathrm{T}})$ associated with a central mass M. A point source is more efficient at accelerating particles to relativistic bulk velocities than a source subtending a finite solid angle Ω, since, in the radiation field of the latter, particles with $\gamma \gtrsim \Omega^{-1/2}$ will experience a net drag force (from aberrated photons) rather than an acceleration. In the spirit of maximizing the potential efficiency and the point-source approximation, we will also ignore the effects of gravity, and assume that the plasma and photons both move in a flat space. This is justified if $M/R_0 = 1/\omega_0 \ll \gamma_{\mathrm{bf}} - 1 \lesssim \omega_0$, where γ_{bf} is the final bulk γ of the plasma and the lower limit is set by the requirement that the plasma have a final velocity well above the escape velocity, while the upper limit is the limit to the point-source approximation set by the curvature of photon geodesics.

If the evolution of the distribution function $f(t, \mathbf{x}, \boldsymbol{\xi})$ of particles in the cage is determined by the radiation force and interparticle collisions which conserve four-momentum ξ^μ, then by taking moments of the Boltzman equation, one arrives at the manifestly covariant equation of bulk motion

$$T^{\mu\nu}_{\mathrm{p},\nu} = \int \frac{dp^\mu}{d\tau} f \frac{d^3\xi}{\xi^0} \equiv \mathscr{F}^\mu , \tag{6}$$

where $dp^\mu/d\tau$ is given by (2) with u^α replaced by ξ^α and $T^{\mu\nu}_{\mathrm{p}} = m_{\mathrm{s}} \int \xi^\mu \xi^\nu f d^3\xi/\xi^0$ is the stress–energy tensor of the particle-fluid. If we suppose that the collisions are sufficiently numerous that there always exists a frame S' with four-velocity u^μ_{b} in which f is isotropic, then we may obtain $\mathscr{F}^\mu$ in an arbitrary frame by Lorentz transformation of $\mathscr{F}^{\mu'}$, given by (4) and (5). If the fluid is a perfect fluid (at rest in S'), then $T^{\mu\nu}_{\mathrm{p}} = (\rho + p) u^\mu_{\mathrm{b}} u^\nu_{\mathrm{b}} + p\eta^{\mu\nu}$, and integrating (6) over the four-volume swept out by the cage in an infinitesimal time interval, observing that the particles do not carry any momentum out of the cage, we obtain the equations of radial motion away from the source:

$$\partial/\partial t\,[(\rho + p)\gamma_{\mathrm{b}} - p/\gamma_{\mathrm{b}}] = \mathscr{F}^{0'} + \beta_{\mathrm{b}} \mathscr{F}^{1'} , \tag{7}$$

$$\partial/\partial t\,[(\rho + p)\gamma_{\mathrm{b}}\beta_{\mathrm{b}}] = \mathscr{F}^{1'} + \beta_{\mathrm{b}} \mathscr{F}^{0'} . \tag{8}$$

Introducing $l = L/L_{\mathrm{Edd}}$, $\omega = R/M$, and using $\beta_{\mathrm{b}} = dR/dt$, the equations of motion for 'dust'

($p \ll \rho$) become:

$$\frac{d\gamma_b}{d\omega} = \frac{l}{\omega^2}\frac{m_p}{m_e}(e + \langle\gamma_r\rangle)^{-1}\frac{1-\beta_b}{1+\beta_b}\left(1 + \frac{2}{3}\langle\gamma_r^2\beta_r^2\rangle\right), \tag{9}$$

$$\frac{d\langle\gamma_r\rangle}{d\omega} = -\frac{l}{\omega^2}\frac{m_p}{m_e}\frac{1-\beta_b}{1+\beta_b}\frac{1}{\gamma_b\beta_b}\frac{4}{3}\langle\gamma_r^2\beta_r^2\rangle, \tag{10}$$

while those for a relativistic perfect gas ($p = 1/3\rho$) are

$$\frac{d\gamma_b}{d\omega} = \frac{l}{\omega^2}\frac{m_p}{m_e}(\rho + \langle\gamma_r\rangle)^{-1}\frac{3}{4}\left(1 - \frac{1}{3}\beta_b^2\right)^{-1}\frac{1-\beta_b}{1+\beta_b}\left[\left(1 + \frac{2}{3}\langle\gamma_r^2\beta_r^2\rangle\right) + \frac{4}{9}\beta_b\langle\gamma_r^2\beta_r^2\rangle\right], \tag{11}$$

$$\frac{d\langle\gamma_r\rangle}{d\omega} = -\frac{l}{\omega^2}\frac{m_p}{m_e}\frac{3}{4}\left(1 - \frac{1}{3}\beta_b^2\right)^{-1}\frac{1}{\gamma_b\beta_b}\frac{1-\beta_b}{1+\beta_b}\left[\left(\frac{4}{3} - \frac{1}{3}\beta_b^2\right)\frac{4}{3}\langle\gamma_r^2\beta_r^2\rangle + \frac{1}{3}\beta_b\left(1 + \frac{2}{3}\langle\gamma_r^2\beta_r^2\rangle\right)\right], \tag{12}$$

where, in deriving (9)–(12), we have put $\rho = \rho_i + n_s m_e\langle\gamma_r\rangle \equiv n_s m_e(e + \langle\gamma_r\rangle)$ and assumed that the scatterers do not share their energy with the other particles present in the plasma (i.e. the electrons cool independently of the ions). If they remain in equipartition, the equations of motion are identical to those obtained from (9)–(12) by replacing l by $(1 + n_i/n_s)^{-1}l$ and e by $(m_i/m_e - 1)(1 + n_s/n_i)^{-1}$, so that all the results obtained below will apply equally to an equipartition plasma, provided these substitutions are made.

To make further progress we must make some assumption about the evolution of the scatterer's energy distribution since equations (9)–(12) *depend* on $\langle\gamma_r^2\rangle$ but *give* us the rate of change of $\langle\gamma_r\rangle$. Compton cooling affects high-energy particles the most; so in the absence of energy–exchange between scatterers, the distribution would narrow as its mean decreased. On the other hand, the processes that are required to keep the momentum distribution isotropic will probably exchange energy too. For simplicity, we assume that these keep $\langle\gamma_r^2\rangle - 1 = f(\langle\gamma_r\rangle^2 - 1)$, where f is a constant ($f = 1$ for a mono-energetic distribution). Approximate global solutions can be obtained by matching the solution of (11) and (12) for $x \equiv \langle\gamma_r\rangle > 1 + e$ on to that of (9) and (10) for $x < 1 + e$. Defining $\psi = (lf/\omega_0)(m_p/m_e)$, the solutions for $\omega \to \infty$ have simple expressions in the following regimes:

$x_f \simeq 0.07\,\psi^{-1}[\psi x_0]^{15/22}$ and $\gamma_{bf} \simeq 0.5[\psi x_0]^{5/22}$

when

$$x_0 \gg \max[1 + e, 2\psi^{-1}, 150(1+e)^{22/15}\psi^{7/15}] \equiv M; \tag{13}$$

$x_f \simeq 1 + (1+e)\,[\![1 + {}^1\!/\!_3(1+e)\psi[40(1+e)/x_0]^{15/7}]\!]^{-1}$

and

$\gamma_{bf} \simeq [\![{}^3\!/\!_4\,\psi/f(1+e) + \{1 + [x_0/40(1+e)]^{5/7}\}^3]\!]^{1/3}$

when

$$40(1+e) \ll x_0 \ll M \quad \text{or} \quad 1 \ll x_0 \ll 1 + e \gg \psi/f; \tag{14}$$

$x_f \simeq 1 + ({}^3\!/\!_8)(x_0/e)\psi^{-1}$ and $\gamma_{bf} \simeq 1 + {}^1\!/\!_8(x_0/e)^2$

when

$$^8/_3 e\psi \ll x_0 \ll 1 + e \gg {}^3/_8 \psi^{-1}; \tag{15}$$

$$x_f \simeq x_0 \quad \text{and} \quad \gamma_{bf} \simeq 1 + (1/2)\psi x_0$$

when

$$1 + e \ll x_0 \ll (1/2)\psi^{-1}. \tag{16}$$

Note that during the rocket phase when $x \gg 1$, the acceleration is always inefficient, since then (9)–(12) give the strict inequality $\gamma_b < [(e + x_0)/(e + x)]^{5/7}$.

In the conditions of (13) the relativistic scatterers provide most of the inertia; cooling and acceleration to relativistic velocities occur on the same time-scales, so that when $\gamma_b \gg 1$, $\gamma_{bf}/\gamma_{b0} = 0.27\,(x_0/x_f)^{5/7}$. When the terms on the right of the square brackets in (14) dominate, the plasma accelerates and cools until the ions begin to dominate the inertia and the acceleration is halted. When the left-hand terms dominate, the plasma cools to $\langle\gamma_r\rangle \sim 1$ before it has been accelerated to a relativistic bulk velocity; the acceleration then takes place exactly as it would for a completely cold plasma (or, with $e = 0$, as it would for free scatterers started from rest). (For comparison, the solution to equation (9) when the γ_r distribution is fixed (either because the plasma is perfectly cold, $\langle\gamma_r\rangle = 1$, or because there is a heat source) is

$$\frac{2}{3}[(\gamma_{bf}^3 - 1) + (\gamma_{bf}^2 - 1)^{3/2}] - (\gamma_{bf} - 1) = \frac{l}{\omega_0}\frac{m_p}{m_e}\frac{(2/3)\langle\gamma_r^2\rangle + (1/3)}{e + \langle\gamma_r\rangle}$$

which is the same as (14) when $\langle\gamma_r\rangle = 1$ and the right hand side is $\gg 1$.) In the conditions of (15), the inertia of the ions prevents acceleration of the bulk plasma while keeping it near the source long enough for the scatterers to cool, while in the conditions of (16), the source is so weak that the plasma is neither cooled nor accelerated significantly (the final bulk velocity given can also be computed by Newtonian mechanics). All other choices of initial conditions lead to $\gamma_{bf} \sim x_f \sim 1$.

Table 1 displays the various regimes for some conditions of astrophysical interest. Assuming the plasma to be initially a few Schwarzschild radii from the source, columns 1–4 correspond respectively to e^+–e^- and e^-–p plasmas around a source operating near the (non-relativistic proton) Eddington limit, and e^+–e^- and e^-–p plasmas around a source operating at $\sim 10^{-4}$ of

Table 1. The domains of applicability for the asymptotic results of equations (13)–(16). We have set $f = 1$, appropriate for a truly relativistic plasma, but not for a largely non-relativistic one with a high-energy non-thermal 'tail'. Columns 1 and 3 apply to e^-–e^+ plasmas, 2 and 4 to e^-–p plasmas, 1 and 2 to starting points near a source emitting the Eddington luminosity and 3 and 4 to starting points far from such a source or near a highly sub-Eddington source.

	(1)	(2)	(3)	(4)
Appropriate equation	$e \lesssim 1, l/\omega_0 = 10^{-1}$	$e = 1800, l/\omega_0 = 10^{-1}$	$e \lesssim 1, l/\omega_0 = 3\times10^{-5}$	$e = 1800, l/\omega_0 = 3\times10^{-5}$
(13)	$x_0 > 2\times10^3$	$x_0 > 10^8$	$x_0 > 40$	$x_0 > 2\times10^6$
(14)	$1 < x_0 < 2\times10^3$	$7\times10^4 < x_0 < 10^8$	never	$7\times10^4 < x_0 < 2\times10^6$
(15)	never	never	never	$260 < x_0 < 1800$
(16)	never	never	$2 < x_0 < 20$	never
$\gamma_{bf} \simeq x_f \simeq 1$	never	$x_0 < 7\times10^4$	$x_0 < 2$	$x_0 < 260$

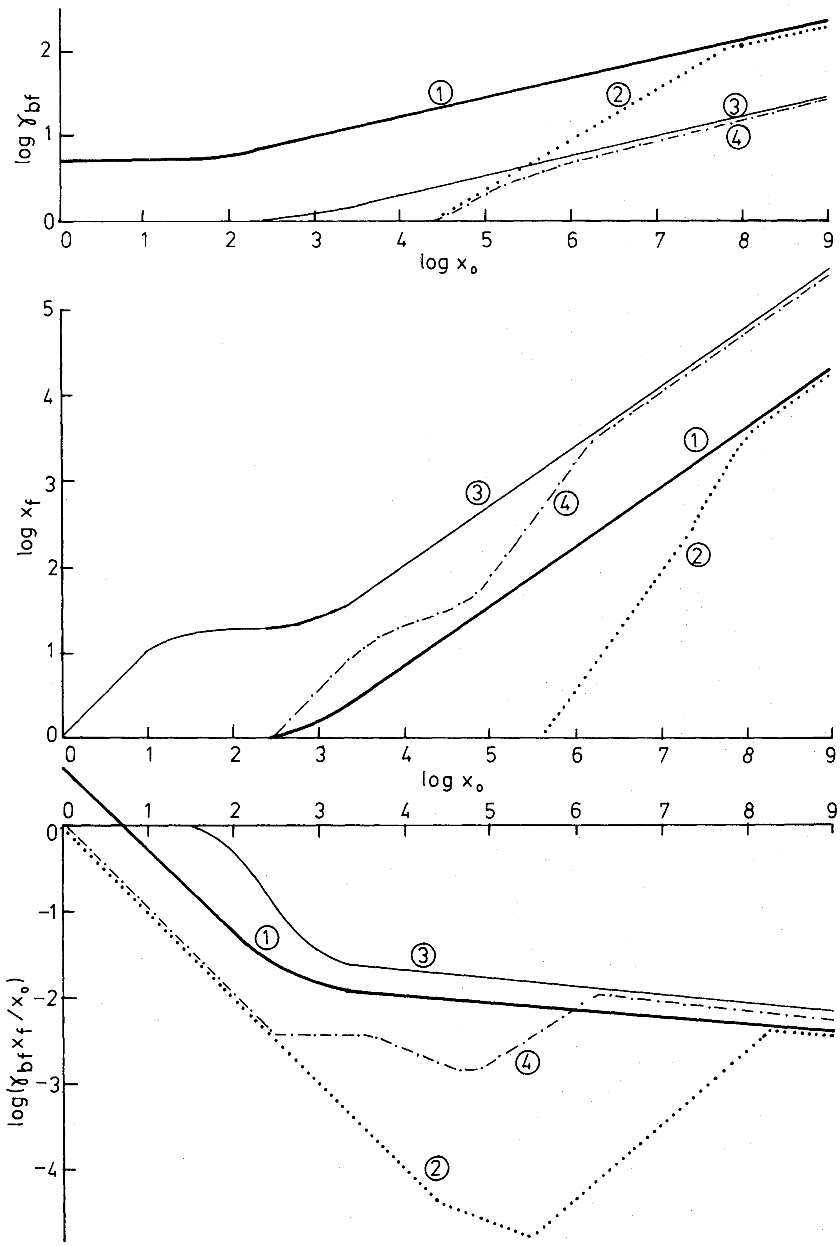

Figure 1. Properties of the plasma after acceleration, as a function of its scatterer's initial average energy per unit rest mass $\langle \gamma_r \rangle \equiv x_0$, for the four representative cases shown in Table 1, whose column numbers correspond to the numbers on the lines: (a) final $\gamma_b = (1 - \beta_b^2)^{-1/2}$ of the bulk motion; (b) final $\langle \gamma_r \rangle \equiv x_f$; (c) net 'efficiency' $\equiv$ (final total energy)/(initial total energy).

the Eddington limit (perhaps typical of low-luminosity radio sources like M87). Fig. 1 shows, for the same four conditions, the final γ_b, $\langle \gamma_r \rangle \equiv x$, and the 'efficiency' as measured in the source frame $(\gamma_{bf} x_f)/x_0$ = (final energy of plasma)/(initial energy of plasma). The latter can be > 1 if most of the acceleration comes from ordinary radiation pressure rather than the rocket effect. Rapid cooling, redshifts, and flux factors all conspire to make γ_{bf} increase only very slowly as x_0 is raised and, except in the extreme case (1), γ_{bf}'s greater than a few are produced with efficiencies less than a few per cent. The rest of the plasma's energy is given to the scattered photons, so the luminosity associated with bulk outflow in a source powered by a Compton rocket can only be a small fraction of the radiated luminosity.

4 The real world: expansion, finite source size, Klein–Nishina scattering, and the origin of the plasma

As Fig. 1 shows, even in the best of circumstances the Compton rocket is a very inefficient way to convert the energy of random internal motions into directed outflow. We now examine the effects of realistic complications on the efficiency and operation of the rocket.

The most obvious defect of our analysis is the assumption that the scattering plasma is confined to a cage. Without such confinement the plasma will cool by expansion, converting random energy into bulk with an efficiency which, in the absence of any source of radiation, is 100 per cent! If $(m_p/m_e)lx_0/\omega_0 \ll 1$, the effects of radiation will be negligible and expansion will determine the dynamics. If $(m_p/m_e)lx_0/\omega_0 \gg 1$, Compton cooling and acceleration will dominate initially and, in driving the plasma from $R = R_0$ to $R = 2R_0$, will set up γ_{bf} and x_f as given in Section 3. Thereafter Compton processes will be unimportant, and if $x_f > 1 + e$, the plasma will expand adiabatically (perhaps confined to a channel) keeping $(x + e)\gamma_b = x_f\gamma_{bf}$, so that the final state is $x = 1$, $\gamma_b = (x_f\gamma_{bf})/(1 + e)$. To within factors of order unity, the net efficiency is thus the same as that of the Compton process alone.

On the other hand, a real source of finite size $\sim SM(S \gtrsim 2!)$ will subtend at $R = \omega M$ an angle $\sim S/\omega$, so that, when $\gamma_b \sim \omega/S$, momentum transfer to the plasma stops, though loss of energy continues. If the γ_{bf} computed in Section 3 is $\gtrsim \omega_0/S$, the true sequence of events is as follows: the plasma is driven by radiation pressure until (at some $R_0 \lesssim R \lesssim 2R_0$) $\gamma_b \sim \omega_0/S$ and $x = x_1$ (given by equations 9–12). It then cools very rapidly (still remaining at $R \lesssim 2R_0$) until

$$x = x_2 \equiv x_1(1 + g)/(1 + gx_1); \quad g \equiv f/3(m_p/m_e)(l/\omega_0)(S/\omega_0)^3. \tag{17}$$

If $1/x_1 < g < 1 + e$, acceleration will stop when the cooling described by (17) has taken place, and the plasma will continue to coast with $\gamma_{bf} \simeq \omega_0/S$. If $g > 1 + e$, then $x_2 \simeq 1$ and $\gamma_{bf} \simeq [(l/S)m_p/(m_e(1 + e))]^{1/4}$. Note that, if $g < 1/x_1$, γ_b is always less than ω_0/S and the effects of finite source size need never have been considered, while, if $g > 1/x_1$, the final γ_b is always less than it would have been with a point source of the same luminosity.

The problem will be made still more difficult if the internal motions are extremely relativistic and the Thomson scattering approximation breaks down, for then the radiation force becomes a complicated function of γ_b, γ_r and the source spectrum. Fig. 1 illustrates the problem – to obtain $\gamma_{bf} \sim 10$ we need $x_0 \sim 10^6$, and, at these particle energies, even optical photons are above the Klein–Nishina limit. Because $\sigma_\nu \propto \nu^{-1}$ above this limit, for typical source spectra one can usually ignore scattering of photons with $h\nu > m_e/\gamma$, and equation (2) will still be approximately valid if, working in the particle's rest frame, we replace $T^{\alpha\beta}$ by

$$\int_0^{m_e/h} d\nu' dT^{\alpha\beta}/d\nu'.$$

For example, if $F_\nu \propto \nu^{-\alpha}$, low energy Thomson-scattered photons determine the motion if $\alpha > 1$, while if $0 < \alpha < 1$ photons with $h\nu' = m_e$ (i.e. $h\nu \sim m_e/\gamma$) are the most important. Only if $\alpha < 0$ do photons scattered above the Klein–Nishina limit significantly affect the dynamics. Compare 3C 273 which has $F_\nu \propto \nu^{-1}$ for $10^{12} < \nu < 10^{24}$ Hz (Ulrich 1981; there is, however, no direct evidence that any of this radiation comes from the region a few Schwarzschild radii across where radiation pressure could be important, since the flux variations at ratio, optical and X-ray frequencies are all on time-scales of months, and it is quite possible that the X-rays are low frequency photons Compton-scattered by the same electrons responsible for the self-absorbed synchrotron emission – presumably in a region ~ 1 pc across). If an appreciable fraction of the source's luminosity *is* at frequencies near $m_e \gamma_b/(h\langle\gamma_r\rangle)$ (e.g. $0 < \alpha < 1$), then the efficiency of the rocket is markedly diminished. This is because the effective flux seen by the (highly scattering) particles moving towards the source is decreased, and that seen by those (less scattering) moving away is increased, thus reducing the asymmetry which drives the rocket.

Finally, we examine briefly the two assumptions central to this analysis: (1) that the scatterers behave like a fluid and exchange momenta rapidly enough to keep an isotropic distribution, and (2) that the plasma exists. The condition that the plasma be optically thin guarantees that it be fully collisionless. Just as in cosmic-ray physics where two-body collisions are also insufficient to couple the particles, collective plasma instabilities are the only possible mechanisms of isotropization. These seem to be very effective in the interstellar medium (though *not* on the much shorter scales associated with the interplanetary plasma), and extrapolation of the conventional rules of thumb suggest that they can be rapid enough at the densities and scales of interest to justify the assumption of isotropy. They might, unfortunately, also generate magnetic fields of such a strength that synchrotron losses would cool the plasma faster than the Compton rocket.

An interesting property of relativistic plasma is that all wave velocities approach c, so that particles scattered by travelling fluctuations experience energy changes $O(1)$, and stochastic acceleration is highly efficient. As explained below, this may cause the 'rocket' effect to be important even if there is no obvious source of ultra-relativistic plasma. In accretion into a relativistic potential well, only $\sim \epsilon$ GeV nucleon^{-1} (where ϵ the efficiency $\lesssim 0.1$) is available for pair production, acceleration of particles and generation of the postulated luminosity, so the globally averaged γ_r must be $< 2000\epsilon/(1+2N_+)$, where N_+ is the number of pairs created per incident nucleon. The Compton (or any other!) rocket cannot therefore lead to even mildly relativistic bulk motion unless the accretion energy is somehow concentrated in a tiny spatially or dynamically isolated fraction of the matter present ($\gtrsim 10^5$ accreted nucleons giving all their energy to the e^- of a single e–p pair or to a few e^+–e^- pairs). In this context, observe that the rocket effect can occur even if the majority of plasma particles are only mildly or even non-relativistic: the force goes as $\langle\gamma_r^2\rangle$ and it is possible for a few high-energy particles unimportant in number $\int dn$ or in energy $\int \gamma_r\, dn$ to determine $\int \gamma_r^2\, dn$. For example, if the differential energy spectrum $dn(\gamma) \propto \gamma^{-p}$ for $\gamma_m < \gamma < \gamma_M$, then for $2 < p < 3$

$$\langle\gamma^2\rangle = \frac{p-1}{3-p}\gamma_m^2\,(\gamma_M/\gamma_m)^{3-p}$$

while

$$\langle\gamma\rangle = \frac{p-1}{p-2}\gamma_m\,.$$

(This corresponds to a huge value of the parameter f introduced in Section 3.) Such energy spectra do seem to occur in cosmical plasmas ($2 < p < 3$ corresponds to a synchrotron spectral index $0.5 < \alpha < 1$!), and may, for example, be produced by stochastic acceleration (*ut supra*) or if particles are repeatedly scattered across a strong shock: simple models of non-relativistic shocks in media in which relativistic particles do not provide most of the pressure give $p = (r+2)/(r-1)$, where r is the compression ratio of the shock (Blandford 1979 and earlier references therein; Peacock 1981 treats relativistic shocks, and finds similar but slightly flatter spectra). Thus $3 > p > 2$ is expected if $2.5 < r < 4$ (the maximum possible compression in an adiabatic shock if $c_p/c_v = 5/3$). That a large fraction of the total accretion energy may be cycled through such particles in a high-energy tail is suggested by the observations of the galactic 'nanoquasar' Cyg X-3 (Hjellming 1978; Neshpor *et al.* 1979; Vestrand & Eichler 1979; Danaher *et al.* 1981) which may have a luminosity in 10^{13} eV γ-rays comparable to its total luminosity in the radio through X-ray band! If these particles are electrons or positrons (rather than protons) and if they are not dynamically coupled to too much deadweight, the rocket effect could give them a relativistic bulk velocity, provided that synchrotron losses do not exceed Compton losses. Observe that in the regime of equation (13), for a given internal energy, an increase of f increases γ_{bf}, but decreases the efficiency $x_f \gamma_{bf}/x_0$.

5 Conclusion

Lest our pessimistic tone make the reader depart with the feeling that the effects we have described are rarely of consequence, we close with a summary of situations in which they are important. First, the structure of radiation-pressure supported (thick) accretion discs can be dramatically altered if some of their electrons are relativistic (equation 5: the effective radiation pressure is increased by a factor of 10^4 if the electrons are in equipartition with 50-MeV protons, or by an unspecified, but similarly large factor if most of the electrons are non-relativistic, but a few in a 'non-thermal tail' make $\langle \gamma_r^2 \rangle \gg 1$). Secondly, if an ultra-relativistic free plasma is formed near a luminous compact object (perhaps by electrodynamic processes in the magnetosphere – Blandford & Znajek 1977; Burns 1980) and if internal cooling is unimportant, the rocket effect will convert a very small fraction of the plasma's random internal energy into bulk kinetic energy (Fig. 1, equations (13)–(16), and warnings in Section 4). Likewise a region around a particle-accelerating strong shock in which $\langle \gamma_r^2 \rangle \gg 1$ will feel a much increased radiation pressue which may affect the dynamics. Since catastrophic Compton cooling is an inseparable companion to the rocket effect, any energy flux associated with accelerated bulk motion is always overshadowed by the much larger energy flux carried by up-scattered photons. Conversely, if the efficiency of a hydrodynamic or electrodynamic scheme for producing plasma beams is threatened by the Compton cooling of its relativistic particles, one should remember that the rocket effect can (in the conditions described in Section 4) return some of the 'loss' as energy of bulk motion. O'Dell's 'Compton rocket' might therefore more appropriately be christened the 'Compton afterburner'.

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